

Probing hybrid modified gravity by stellar motion around Galactic Centre

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Abstract

We consider possible signatures for the so called *hybrid gravity* within the Galactic Central Parsec. This modified theory of gravity consists of a superposition of the metric Einstein-Hilbert Lagrangian with an $f(R)$ term constructed *à la Palatini* and can be easily reduced to an equivalent scalar-tensor theory. Such an approach is introduced in order to cure the shortcomings related to $f(R)$ gravity, in general formulated either in metric or in metric-affine frameworks. Hybrid gravity allows to disentangle the further gravitational degrees of freedom with respect to those of standard General Relativity. The present analysis is based on the S2 star orbital precession around the massive compact dark object at the Galactic Centre where the simulated orbits in hybrid modified gravity are compared with astronomical observations. These simulations result with constraints on the range of hybrid gravity interaction parameter ϕ_0 , showing that in the case of S2 star it is between -0.0009 and -0.0002 . At the same time, we are also able to obtain the constraints on the effective mass parameter m_ϕ , and found that it is between -0.0034 and -0.0025 AU^{-1} for S2 star. Furthermore, the hybrid gravity potential induces precession of S2 star orbit in the same direction as General Relativity. In previous papers, we considered other types of extended gravities, like metric power law $f(R) \propto R^n$ gravity, inducing Yukawa and Sanders-like gravitational potentials, but it seems that hybrid gravity is the best among these models to explain different gravitational phenomena at different astronomical scales.

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1. Introduction

The existence of different anomalous astrophysical and cosmological phenomena like the cosmic acceleration, the dynamics of galaxies and gas in clusters of galaxies, the galactic rotation curves, etc. recently boosted the growth of several long-range modifications of the usual laws of gravitation. These mentioned phenomena did not find satisfactory explanations in terms of the standard Newton-Einstein gravitational physics, unless exotic and still undetected forms of matter-energy are postulated: dark matter and dark energy. A recent approach is to try to explain these phenomena without using new material ingredients like dark matter and dark energy, but using well-motivated generalization and extensions of General Relativity (GR). Several alternative gravity theories have

been proposed (see e.g. [1, 2, 3, 4, 5, 6, 7] for reviews), such as: MOND [8], scalar-tensor [9, 10, 11, 12], conformal [13, 14], Yukawa-like corrected gravity theories [15, 16, 17, 18], theories of "massive gravity" [19, 20, 21, 22, 23, 24, 25]. Alternative approaches to Newtonian gravity in the framework of the weak field limit [26] of fourth-order gravity theory have been proposed and constraints on these theories have been discussed [27, 28, 31, 32, 30, 33, 29, 34, 35, 36, 37, 38].

The philosophy is to search for alternative form of gravity, i.e. of the Einstein-Hilbert theory, so that such modifications could naturally explain some astrophysical and cosmological phenomena without invoking the presence of new material ingredients that, at the present state of the art, seem hard to be detected. Besides, this extended approach can be connected to effective theories that emerge both from the quantization on curved spacetimes and from several unification schemes [2, 3, 4].

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The simplest extension of the Einstein-Hilbert action is based on straightforward generalizations of the Einstein theory where the gravitational action (the Einstein-Hilbert action) is assumed to be linear in the Ricci curvature scalar R . If this action consists in modifying the geometric part considering a generic function $f(R)$, we get so called $f(R)$ gravity which was firstly proposed in 1970 by Buchdahl [39]. Generally, the most serious problem of $f(R)$ theories is that these theories cannot easily pass the standard Solar System tests [40, 41]. However, there exists some classes of them that can solve this problem [42]. It can be shown that $f(R)$ theories, in principle, could explain the evolution of the Universe, from a matter dominated early epoch up to the present, late-time self accelerating phase. Several debates are open in this perspective [43, 44, 46, 45] but the crucial point is that suitable self-consistent model can be achieved. $f(R)$ theories have also been studied in the Palatini approach, where the metric and the connection are regarded as independent fields [47]. Metric and Palatini approaches are certainly equivalent in the context of GR, i.e., in the case of the linear Einstein-Hilbert action. This is not so for extended gravities. The Palatini variational approach leads to second order differential field equations, while the resulting field equations in the metric approach are fourth order coupled differential equations. These differences also extend to the observational aspects.

A novel approach, that consists of adding to the metric Einstein-Hilbert Lagrangian an $f(R)$ term constructed within the framework of the Palatini formalism, was recently proposed [48, 49, 50]. The aim of this formulation is twofold: from one side, one wants to describe the extra gravitational budget in metric-affine formalism, from the other side, one wants to cure the shortcomings emerging in $f(R)$ gravity both in metric and Palatini formulations. In particular, hybrid gravity allows to disentangle the metric and the geodesic structures pointing out that further degrees of freedom coming from $f(R)$ can be recast as an auxiliary scalar field. In such a case, problems related to the Brans-Dicke-like representation of $f(R)$ gravity in terms of scalar-tensor theory (the so called O’Hanlon transformation) are immediately avoided (see [50] for details and the discussion in Sec. 2). Due to this feature, the scalar-tensor representation of hybrid gravity results preferable with respect to other scalar-tensor representations of gravitational interaction. As byproducts, the appearance of ghosts is avoided and the correct weak field limit of $f(R)$ gravity with respect to GR is recovered. Furthermore, several issues related to the galactic dynamics, the formulation of the virial theorem in alternative gravity, the

dark energy behavior seem to be better addressed than in $f(R)$ gravity considered in both metric or Palatini formulations. In summary, the hybrid metric-Palatini theory opens up new possibilities to approach, in the same theoretical framework, the problems of both dark energy and dark matter disentangling the extra degrees of freedom of gravitational field with respect to GR. For a brief review on the hybrid metric-Palatini theory, we refer the reader to [51].

In this perspective, star dynamics around the Galactic Centre could be a useful test bed to probe the effective gravitational potentials coming from the theory. In particular, S-stars are the young bright stars which move around the centre of our Galaxy [52, 56, 54, 55, 53, 57] where the compact radio source Sgr A* is located. For more details about S2 star see references [58, 57]. There are some observational indications that the orbits of some of them, like S2, could deviate from the Keplerian case [54, 59], but the current astrometric limit is not sufficient to unambiguously confirm such a claim [36, 60].

Here we study a possible application of hybrid modified gravity within Galactic Central Parsec, in order to explain the observed precession of orbits of S-stars. This paper is a continuation of previous studies where we considered different extended gravities, such as power law $f(R)$ gravity [29, 38], $f(R, \phi)$ gravity implying Yukawa and Sanders-like gravitational potentials in the weak field limit [36, 37]. Results obtained using hybrid gravity point out that, very likely, such a theory is the best candidate among those considered to explain (within the same picture) different gravitational phenomena at different astronomical scales.

More details about hybrid gravity you can find in [47, 48, 50, 51]. It is shown in [50, 51] that this type of modified gravity is coherently addressing the Solar System issues, and motivations for addressing them are discussed in details in [51].

The modified theory of gravity needs to be constrained at different scales: at laboratory distances, at Solar system, at galaxies, at galactic clusters and at cosmological scales. Obtaining constraints at any of these scales is a fundamental issue to select or rule out models. In particular, it is important to investigate gravity in the vicinity of very massive compact objects because the environment around these objects is drastically different from that in the Solar System framework. The S2 star orbit is a unique opportunity to test gravity at the sub-parsec scale of a few thousand AU. For example, gravity is relatively well constrained at short ranges (especially at sub-mm scale) by experimental tests, however for long ranges further tests are still needed (see

Figures 9 and 10 from [61] for different ranges). It is worth stressing that a phenomenological approach can be useful in this context. In particular, the motion of S2-star is a suitable tool to test alternative gravity. For the reasons that we will discuss in detail below, hybrid gravity is a reliable paradigm to describe gravitational interaction without considering dark energy and dark matter. Specifically, the massive compact object inside the Galactic Center is surrounded by a matter distribution and deviations of S2-star motion from the Keplerian orbit are observed in detail. These deviations can be triggered both by the masses of the surrounding bodies and by the strong field regime at the Galactic Center. This peculiar situation constitutes a formidable opportunity to test theories of gravity. However, it is important to stress that numerical results reported here by comparing models with astronomical observations, represent only upper bounds for the precession angle on the deviation from GR. More accurate studies will be necessary in future work to better constrain dynamics around the Galactic Center.

The present paper is organized as follows: in Sec. 2 we sketch the theory of hybrid gravity. In Sec. 3 we describe our simulations of stellar orbits in the gravitational potential derived in the weak field limit of hybrid gravity and we describe the fitting procedure to simulate orbits with respect to astrometric observations of S2 star. Results are presented in Sec. 4. Conclusions are drawn in Sec. 5.

2. Hybrid metric-Palatini gravity

In this Section, we present the basic formalism for the hybrid metric-Palatini gravitational theory within the equivalent scalar-tensor representation (we refer the reader to [50, 51, 62, 63] for more details). The $f(R)$ theories are the special limits of the one-parameter class of theories where the scalar field depends solely on the stress energy trace T (Palatini version) or solely on the Ricci curvature R (metric version). Here, we consider a one-parameter class of scalar-tensor theories where the scalar field is given as an algebraic function of the trace of the matter fields and the scalar curvature [62]:

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} \phi R - \frac{D-1}{2(D-2)(\Omega_A - \phi)} (\partial\phi)^2 - V(\phi) \right]. \quad (1)$$

The theories can be parameterized by the constant Ω_A . The limiting values $\Omega_A = 0$ and $\Omega_A \rightarrow \infty$ correspond to scalar-tensor theories with the Brans-Dicke parameter $\omega = -(D-1)/(D-2)$ and $\omega = 0$. These limits reduce to

$f(R)$ gravity in the Palatini and the metric formalism, respectively. For any finite value of Ω_A , its value depends both on matter and curvature. In the limit $\Omega_A \rightarrow \infty$ the propagating mode is given solely by the curvature, $\phi(R, T) \rightarrow \phi(R)$, and in the limit $\Omega_A \rightarrow 0$ solely the matter fields $\phi(R, T) \rightarrow \phi(T)$. In the general case, the field equations are fourth order both in the matter and in the metric derivatives as we will show below.

More specifically, the intermediate theory with $\Omega_A = 1$ and $D = 4$, corresponds to the hybrid metric-Palatini gravity theory proposed in [48, 50], where the action is given by

$$S = \int d^4 x \sqrt{-g} [R + f(\mathcal{R}) + 2\kappa^2 \mathcal{L}_m]. \quad (2)$$

where $\kappa^2 \equiv 8\pi G$, R is the Einstein-Hilbert term, $\mathcal{R} \equiv g^{\mu\nu} \mathcal{R}_{\mu\nu}$ is the Palatini curvature with the independent connection $\hat{\Gamma}_{\mu\nu}^\alpha$ as

$$\mathcal{R} \equiv g^{\mu\nu} \mathcal{R}_{\mu\nu} \equiv g^{\mu\nu} (\hat{\Gamma}_{\mu\nu,\alpha}^\alpha - \hat{\Gamma}_{\mu\alpha,\nu}^\alpha + \hat{\Gamma}_{\alpha\lambda}^\alpha \hat{\Gamma}_{\mu\nu}^\lambda - \hat{\Gamma}_{\mu\lambda}^\alpha \hat{\Gamma}_{\alpha\nu}^\lambda). \quad (3)$$

The Palatini-Ricci tensor $\mathcal{R}_{\mu\nu}$ is

$$\mathcal{R}_{\mu\nu} \equiv \hat{\Gamma}_{\mu\nu,\alpha}^\alpha - \hat{\Gamma}_{\mu\alpha,\nu}^\alpha + \hat{\Gamma}_{\alpha\lambda}^\alpha \hat{\Gamma}_{\mu\nu}^\lambda - \hat{\Gamma}_{\mu\lambda}^\alpha \hat{\Gamma}_{\alpha\nu}^\lambda. \quad (4)$$

Varying the action given with respect to the metric, one obtains the field equations

$$G_{\mu\nu} + F(\mathcal{R}) \mathcal{R}_{\mu\nu} - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (5)$$

where the matter stress-energy tensor is

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta(g^{\mu\nu})}. \quad (6)$$

The independent connection is compatible with the metric $F(\mathcal{R}) g_{\mu\nu}$, conformal to $g_{\mu\nu}$, with the conformal factor given by $F(\mathcal{R}) \equiv df(\mathcal{R})/d\mathcal{R}$. This fact gives

$$\mathcal{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2} \frac{1}{F^2(\mathcal{R})} F(\mathcal{R})_{,\mu} F(\mathcal{R})_{,\nu} - \frac{1}{F(\mathcal{R})} \nabla_\mu F(\mathcal{R})_{,\nu} - \frac{1}{2} \frac{1}{F(\mathcal{R})} g_{\mu\nu} \nabla_\alpha \nabla^\alpha F(\mathcal{R}). \quad (7)$$

The Palatini curvature \mathcal{R} is obtained from the trace of the field equations (5), which is

$$F(\mathcal{R}) \mathcal{R} - 2f(\mathcal{R}) = \kappa^2 T + R \equiv X. \quad (8)$$

\mathcal{R} can be algebraically expressed in terms of X if $f(\mathcal{R})$ is analytic. In other words, the variable X measures how the theory deviates from GR trace equation $R = -\kappa^2 T$.

We can express the field equations (5) in terms of the metric and X as

$$\begin{aligned} G_{\mu\nu} &= \frac{1}{2}f(X)g_{\mu\nu} - F(X)R_{\mu\nu} + F'(X)\nabla_\mu X_{,\nu} \\ &+ \frac{1}{2}\left[F'(X)\nabla_\alpha\nabla^\alpha X + F''(X)(\partial X)^2\right]g_{\mu\nu} \\ &+ \left[F''(X) - \frac{3}{2}\frac{(F'(X))^2}{F(X)}\right]X_{,\mu}X_{,\nu} + \kappa^2 T_{\mu\nu} \end{aligned} \quad (9)$$

being $(\partial X)^2 = X_{,\mu}X^{,\mu}$. The trace of the field equations is now

$$\begin{aligned} F'(X)\nabla_\alpha\nabla^\alpha X + \left[F''(X) - \frac{1}{2}\frac{(F'(X))^2}{F(X)}\right](\partial X)^2 \\ + \frac{1}{3}[X + 2f(X) - F(X)R] = 0, \end{aligned} \quad (10)$$

while the relation between the metric scalar curvature R and the Palatini scalar curvature \mathcal{R} is

$$\mathcal{R}(X) = R + \frac{3}{2}\left[\left(\frac{F'(X)}{F(X)}\right)^2 - 2\frac{\nabla_\alpha\nabla^\alpha F(X)}{F(X)}\right], \quad (11)$$

which can be obtained by contracting Eq. (7). As for pure metric and Palatini cases [4], the action (2) for the hybrid metric-Palatini theory can be recast into a scalar-tensor theory by an auxiliary field A such that

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + f(A) + f_A(\mathcal{R} - A)] + S_m, \quad (12)$$

where $f_A \equiv df/dA$ and S_m is the matter action. Rearranging the terms and defining $\phi \equiv f_A$, $V(\phi) = Af_A - f(A)$, Eq. (12) becomes

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \phi\mathcal{R} - V(\phi)] + S_m. \quad (13)$$

The variation of this action with respect to the metric, the scalar ϕ and the connection leads to the field equations

$$R_{\mu\nu} + \phi\mathcal{R}_{\mu\nu} - \frac{1}{2}(R + \phi\mathcal{R} - V)g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (14)$$

$$\mathcal{R} - V_\phi = 0, \quad (15)$$

$$\hat{\nabla}_\alpha(\sqrt{-g}\phi g^{\mu\nu}) = 0, \quad (16)$$

respectively. The solution of Eq. (16) implies that the independent connection is the Levi-Civita connection of a metric $h_{\mu\nu} = \phi g_{\mu\nu}$, that is we are dealing with a bi-metric theory and $\mathcal{R}_{\mu\nu}$ and $R_{\mu\nu}$ are related by

$$\mathcal{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2\phi^2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi + \frac{1}{2}g_{\mu\nu}\nabla_\alpha\nabla^\alpha\phi\right), \quad (17)$$

which can be used in the action (13) to obtain the following scalar-tensor representation

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[(1 + \phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi) \right] + S_m. \quad (18)$$

We have to stress that, by the substitution $\phi \rightarrow -(\kappa\phi)^2/6$, the action (18) reduces to the case of a conformally coupled scalar field with a self-interaction potential. This redefinition makes the kinetic term in the action (18) the standard one, and the action itself becomes that of a massive scalar-field conformally coupled to the Einstein gravity. Of course, it is not the Brans-Dicke gravity where the scalar field is massless.

As discussed above, in the limit $\Omega_A \rightarrow 0$, the theory (18) becomes the Palatini- $f(\mathcal{R})$ gravity, and in the limit $\Omega_A \rightarrow \infty$ it is the metric $f(R)$ gravity. Apart from these singular cases, any theory with a finite Ω_A is in the "hybrid" regime, which from this point of view provides a unique interpolation between the two a priori completely distinct classes of gravity theories.

Using Eq. (17) and Eq. (15) in Eq. (14), the metric field equations are

$$\begin{aligned} (1 + \phi)R_{\mu\nu} &= \kappa^2 \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right) \\ &+ \frac{1}{2}g_{\mu\nu}(V + \nabla_\alpha\nabla^\alpha\phi) \\ &+ \nabla_\mu\nabla_\nu\phi - \frac{3}{2\phi}\partial_\mu\phi\partial_\nu\phi, \end{aligned} \quad (19)$$

and then the spacetime curvature is sourced by both matter and scalar field. The scalar field equation can be manipulated in two different ways that illustrate how this theory is related with the $w = 0$ and $w = -3/2$ cases, which corresponds to the metric and Palatini scalar-tensor representations of $f(R)$ -gravity [4] respectively. Considering the trace of Eq. (14) with $g^{\mu\nu}$, we find $-R - \phi\mathcal{R} + 2V = \kappa^2 T$, and using Eq. (15), it is

$$2V - \phi V_\phi = \kappa^2 T + R. \quad (20)$$

Similarly as in the Palatini case ($w = -3/2$), this equation says that the field ϕ can be expressed as an algebraic function of the scalar $X \equiv \kappa^2 T + R$, i.e., $\phi = \phi(X)$. In the pure Palatini case, however, ϕ is just a function of T . Therefore the right-hand side of Eq. (19) contains matter terms associated with the trace T , its derivatives, and also the curvature R and its derivatives. In other words, this theory can be seen as a higher-derivative theory in both matter and metric fields. However, such an interpretation can be avoided if R is replaced in Eq. (20) with the relation

$$R = \mathcal{R} + \frac{3}{\phi}\nabla_\mu\nabla^\mu\phi - \frac{3}{2\phi^2}\partial_\mu\phi\partial^\mu\phi \quad (21)$$

together with $\mathcal{R} = V_\phi$. One then finds that the scalar field dynamics is given by a second-order equation that becomes, for $\Omega_A = 1$,

$$-\nabla_\mu \nabla^\mu \phi + \frac{1}{2\phi} \partial_\mu \phi \partial^\mu \phi + \frac{\phi[2V - (1 + \phi)V_\phi]}{3} = \frac{\phi\kappa^2}{3} T, \quad (22)$$

which is a Klein-Gordon equation. This result shows that, unlike in the Palatini case ($w = -3/2$), the scalar field is dynamical. In this sense, the theory is not affected by the microscopic instabilities that arise in Palatini models (see [47] for details).

3. The weak field limit and the fitting procedure

In the weak field limit and far from the sources, the scalar field behaves as $\phi(r) \approx \phi_0 + (2G\phi_0 M/3r)e^{-m_\phi r}$; the effective mass is defined as

$$m_\phi^2 \equiv (2V - V_\phi - \phi(1 + \phi)V_{\phi\phi})/3|_{\phi=\phi_0}, \quad (23)$$

where ϕ_0 is the amplitude of the background value of ϕ . Furthermore V , V_ϕ and $V_{\phi\phi}$ are respectively the potential and its first and the second derivatives with respect to ϕ . The metric perturbations yield

$$\begin{aligned} h_{00}^{(2)}(r) &= \frac{2G_{\text{eff}}M}{r} + \frac{V_0}{1 + \phi_0} \frac{r^2}{6}, \\ h_{ij}^{(2)}(r) &= \left(\frac{2\gamma G_{\text{eff}}M}{r} - \frac{V_0}{1 + \phi_0} \frac{r^2}{6} \right) \delta_{ij}, \end{aligned} \quad (24)$$

where V_0 is the minimum of the potential V . The effective Newton constant G_{eff} and the post-Newtonian parameter γ are defined as

$$\begin{aligned} G_{\text{eff}} &\equiv \frac{G}{1 + \phi_0} [1 - (\phi_0/3)e^{-m_\phi r}], \\ \gamma &\equiv \frac{1 + (\phi_0/3)e^{-m_\phi r}}{1 - (\phi_0/3)e^{-m_\phi r}}. \end{aligned} \quad (25)$$

The coupling of the scalar field to the local system depends on ϕ_0 . If $\phi_0 \ll 1$, then $G_{\text{eff}} \approx G$ and $\gamma \approx 1$ regardless of the value of m_ϕ^2 . This is in contrast with the result obtained in the metric version of $f(R)$ theories. For sufficiently small ϕ_0 , this modified theory allows to pass the Solar System tests, even if the scalar field is very light [51]. According to these considerations, the leading parameters are m_ϕ and ϕ_0 . Their value give both an estimation of the deviation with respect to GR and how the affine contribution (i.e. the Palatini term) is relevant with respect to the metric $f(R)$ gravity. Constraining both of them by observations gives immediately information on the hybrid gravity. Starting from the above

results, the modified gravitational potential can be written in the form:

$$\Phi(r) = -\frac{G}{1 + \phi_0} [1 - (\phi_0/3)e^{-m_\phi r}] M/r. \quad (26)$$

An important remark is necessary at this point. We have not chosen the form of $V(\phi)$ since the only requirement is that the scalar field potential is an analytic function of ϕ . In such a case, the effective mass (23) can be always defined. Clearly, the aim is to derive specific forms of the potential starting from the observations. This means a sort of "inverse scattering procedure" by which the $V(\phi)$ potential can be reconstructed from the observed values of the parameters M , ϕ_0 , m_ϕ and γ .

To this end, let us use eq. (26) to simulate orbits of S2 star in the hybrid modified gravity potential and then we compare the obtained results with the set of S2 star observations obtained by the New Technology Telescope/Very Large Telescope (NTT/VLT). The simulated orbits of S2 star are obtained by numerical integration of equations of motion where the hybrid gravitational potential is adopted, i.e.

$$\dot{\mathbf{r}} = \mathbf{v}, \quad \mu \ddot{\mathbf{r}} = -\nabla \Phi(\mathbf{r}), \quad (27)$$

where μ is the reduced mass in the two-body problem. In that way we obtained the simulated orbit of S2 star around Galactic Centre in the weak field approximation of hybrid gravity where eqs. (24) and (25) stand. Taking into account that $\gamma = \gamma(\phi_0, m_\phi)$, the considered weak field solution depends on the following three parameters: M , ϕ_0 , and m_ϕ . Mass M of the central object can be obtained independently using different observational techniques, such as e.g. virial analysis of the ionized gas in the central parsec [66] (yielding $M = 3 \times 10^6 M_{\text{sun}}$), $M - \sigma$ (mass - bulge velocity dispersion) relationship for the Milky Way [67] (yielding $M = 9.4 \times 10^6 M_{\text{sun}}$) or from Keplerian orbits of S-stars [55] (yielding $M = 4.3 \times 10^6 M_{\text{sun}}$). Since our goal was not to make a new estimate of mass M using hybrid gravity, but instead to study the possible deviations from Keplerian orbit of S2 star which could indicate signatures for hybrid gravity on these scales, we adopted the last of three previously mentioned estimates for mass of the central object ($M = 4.3 \times 10^6 M_{\text{sun}}$), as well as the distance to the S2 star given by [54] ($d_\star = 8.3$ kpc), and constrained only the remaining two free parameters (ϕ_0 , m_ϕ). Parameter ϕ_0 is dimensionless, while m_ϕ is given in AU^{-1} (AU being astronomical unit), so that m_ϕ^{-1} represents a scaling parameter for gravity interaction. Non-zero values of these two parameters, if obtained, would indicate a potential deviation from GR.

In order to obtain the constraints on ϕ_0 and m_ϕ , these two parameters were varied. For each their combination the simulated coordinates x and y and velocity components v_x and v_y of S2 star were calculated. Calculations were performed for each observational epoch and then compared with its corresponding observed positions and velocities. χ^2 between the observed and calculated coordinates of S2 star is minimized using LMDIF1 routine from MINPACK-1 Fortran 77 library which solves the nonlinear least squares problems by a modification of Marquardt-Levenberg algorithm [64] (for more details on fitting procedures see [36]).

4. Results: simulations vs observations

Let us now discuss the numerical simulations that we want to compare with observations in order to select the range of the potential parameters (3.4). As we will see, analysis by hybrid gravity fixes better the observational data than the standard Keplerian analysis.

4.1. Numerical calculation of S2 star orbit and orbital precession

The simulated orbits of S2 star around the central object in hybrid gravity (blue solid line) and in Newtonian gravity (red dashed line) for $\phi_0 = -0.00033$ and $m_\phi = -0.0028$ (left panel), as well as for $\phi_0 = -0.000033$ and $m_\phi = -0.00028$ (right panel) during 5 orbital periods, are presented in Fig. 1. As it can be seen from this figure, hybrid gravity causes the orbital precession in the same direction as GR, but precession angle is much bigger. When both ϕ_0 and m_ϕ are decreased for an order of magnitude, the precession is much smaller (see the right panel of Fig. 1). This analysis also shows that Keplerian orbit is recovered when ϕ_0 and m_ϕ tend to 0.

We calculate orbital precession in hybrid modified gravity potential and results are reported in Fig. 2 as a function of ϕ_0 and m_ϕ . Assuming that the hybrid potential does not differ significantly from Newtonian potential, we derive the perturbed potential as

$$V(r) = \Phi(r) - \Phi_N(r) \quad ; \quad \Phi_N(r) = -\frac{GM}{r} \quad . \quad (28)$$

The obtained perturbing potential is of the form:

$$V(r) = \frac{G}{1 + \phi_0} [1 + (1/3)e^{-m_\phi r}] M\phi_0/r. \quad (29)$$

and it can be used for calculating the precession angle according to Eq. (30) in Ref. [65]:

$$\Delta\theta = \frac{-2L}{GMe^2} \int_{-1}^1 \frac{z \cdot dz}{\sqrt{1-z^2}} \frac{dV(z)}{dz}, \quad (30)$$

where r is related to z via: $r = \frac{L}{1+ez}$. By differentiating the perturbing potential $V(z)$ and substituting its derivative and $(L = a(1-e^2))$ in the above Eq. (30), and taking the same values for orbital elements of S2 star like in Ref. [29] we obtain numerically, for $\phi_0 = -0.00033$ and $m_\phi = -0.0028$, that the precession per orbital period is $3^\circ.26$.

Graphical representation of precession per orbital period for ϕ_0 in the range $[-0.0009, -0.0002]$ and m_ϕ in $[-0.0034, -0.0025]$ is given in the left panel of Fig. 2. As one can see, the pericenter advance (like in GR) is obtained. The precession per orbital period for ϕ_0 in the range $[-0.0004, -0.0002]$ and m_ϕ in $[-0.0029, -0.0027]$ is given in the right panel of Fig. 2.

4.2. Comparison of theoretical results and observations

Let us give some constraints on parameters ϕ_0 and m_ϕ of hybrid gravity potentials according to current available observations of S2 star orbit. However we should note that the present astrometric limit is still not sufficient to definitely confirm that the S2 orbit deviates from the Keplerian one, but there is great probability that it is the case because the astrometric accuracy is constantly improving from around 10 mas during the first part of the observational period, currently reaching less than 0.3 mas (see reference [60]). There are also some recent studies that provide more and more evidence that the orbit of S2 star is not closing (see e.g. Fig. 2 in paper [59]). In this paper we fitted the NTT/VLT astrometric observations of S2 star, which contain a possible indication for orbital precession around the massive compact object at Galactic Centre, in order to constrain the parameters of hybrid gravity potential, since this kind of potential has not been tested at these scales yet. We have to stress that in the reference [55] on page 1092, Fig. 13, authors presented the Keplerian orbit, but in order to obtain it they had to move the position of central point mass. In that way they implicitly assumed orbital precession. In our orbit calculation we do not need to move central point mass in order to get a satisfactory fit.

In fact, our comparison with astronomical observations represents upper bounds for precession angle on deviation from GR. The most probably results for precession are in between these upper bounds and GR results. In future, using more precise astronomical observations, we could obtain more accurate results.

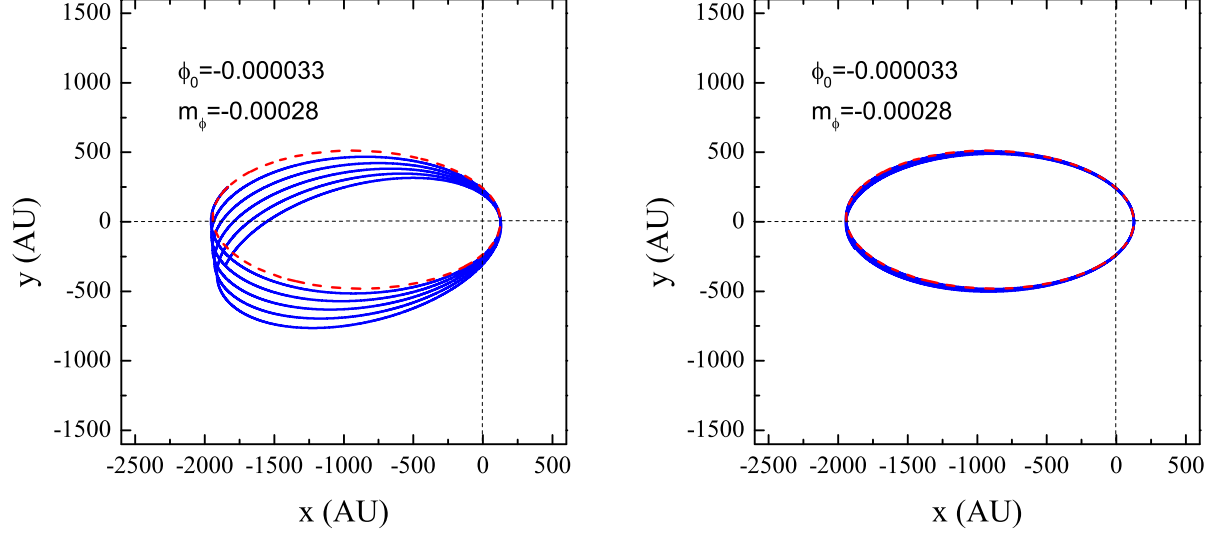


Figure 1: (Color online) Comparisons between the orbit of S2 star in Newtonian gravity (red dashed line) and hybrid gravity during 5 orbital periods (blue solid line) for (left panel) $\phi_0 = -0.000033$ and $m_\phi = -0.00028$, and for (right panel) $\phi_0 = -0.000033$ and $m_\phi = -0.00028$.

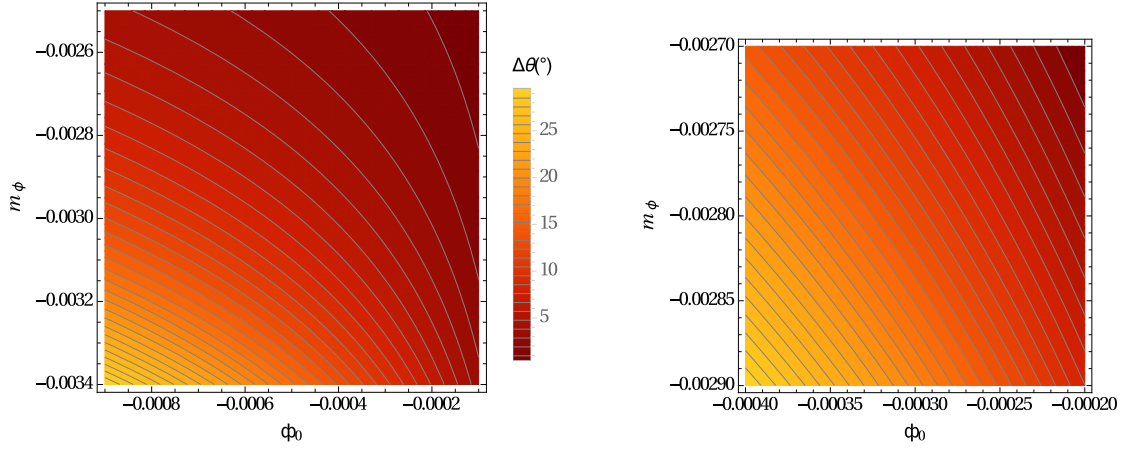


Figure 2: (Color online) The precession per orbital period for ϕ_0 in the range $[-0.0009, -0.0002]$ and m_ϕ in $[-0.0034, -0.0025]$ (left panel), and ϕ_0 in the range $[-0.0004, -0.0002]$ and m_ϕ in $[-0.0029, -0.0027]$ (right panel) in the case of hybrid modified gravity potential. With a decreasing value of angle of precession colors are darker.

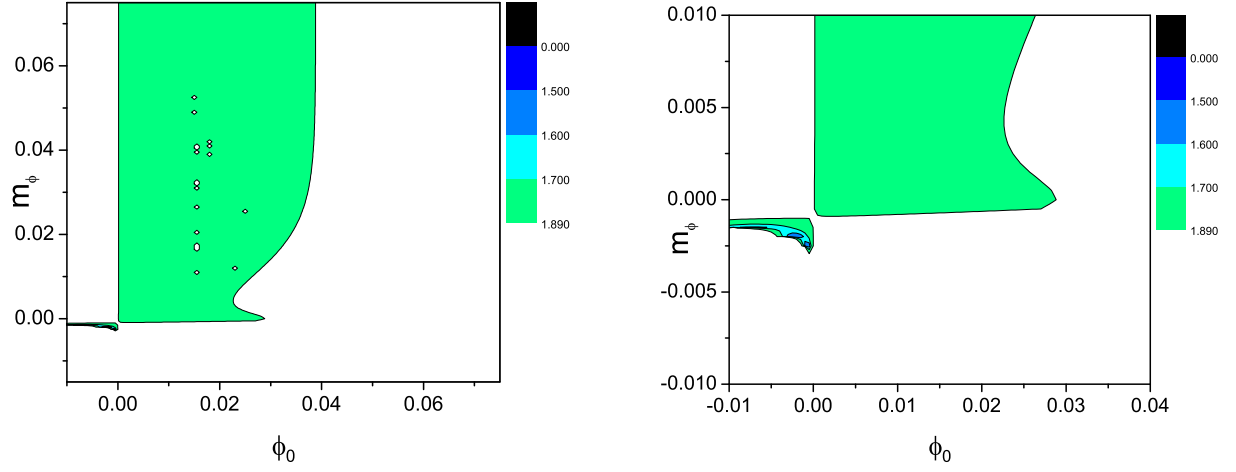


Figure 3: (Color online) The maps of the reduced χ^2 over the $\phi_0 - m_\phi$ parameter space for all simulated orbits of S2 star which give at least the same or better fits than the Keplerian orbits. With a decreasing value of χ^2 (better fit) colors in grey scale are darker. A few contours are presented for specific values of reduced χ^2 given in the figure's legend.

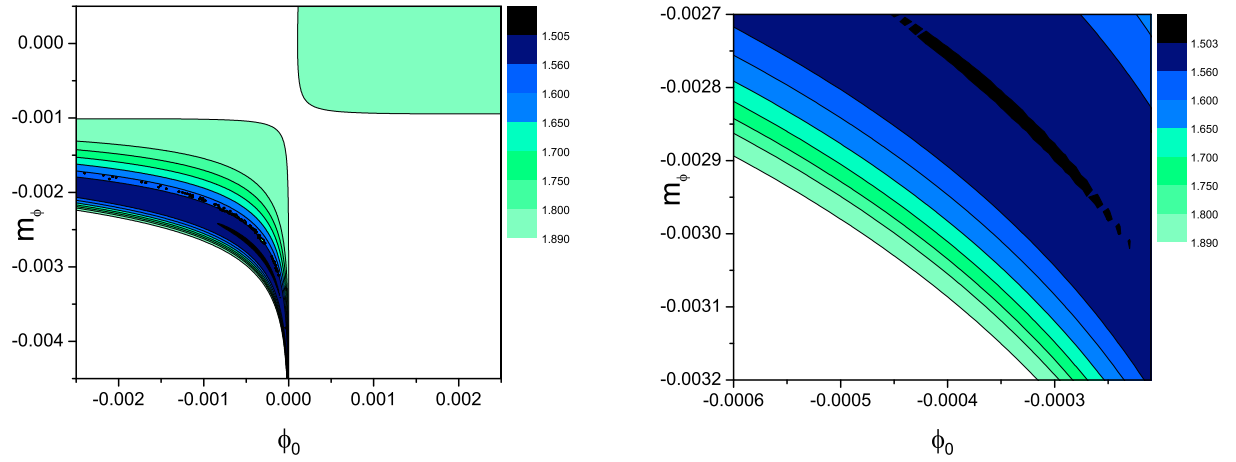


Figure 4: (Color online) The same as in Fig. 3, but for the zoomed range of parameters.

Figs. 3 and 4 present the maps of the reduced χ^2 over the $\phi_0 - m_\phi$ parameter space for all simulated orbits of S2 star which give at least the same or better fits than the Keplerian orbits. These maps are obtained by the same fitting procedure as before. As it can be seen from Figs. 3 and 4, the most probable value for the parameter ϕ_0 in the case of NTT/VLT observations of S2 star is between -0.0009 and -0.0002 and for the parameter m_ϕ is between -0.0034 and -0.0025 (see the darkest regions in Figs. 4). In other words, we obtain reliable constraints on the parameters ϕ_0 and m_ϕ of hybrid modified gravity. The absolute minimum of the reduced χ^2 ($\chi^2 = 1.503$) is obtained for $\phi_0 = -0.00033$ and $m_\phi = -0.0028$, respectively.

We simulated orbits of S2 star around the central object considering both the hybrid gravitational potential and the Newtonian potential. Our analysis shows that the hybrid modified gravity potential induces the precession of S2 star orbit in the same direction of GR. We used these simulated orbits to fit the observed orbits of S2 star. The best fit (according to NTT/VLT data) is obtained for the ϕ_0 from between -0.0009 and -0.0002, and for the m_ϕ between -0.0034 and -0.0025. This range corresponds to scale parameter m_ϕ^{-1} from (1/0.0034) AU to (1/0.0025) AU ($\approx 300 - 400$ AU, i.e. 1.4 – 1.9 mpc) which is comparable to the size of S2 star orbit.

We believe that comparison with astronomical observation is important, and data we used are the best currently published and available. GR predicts that the pericenter of S2 star should advance by $0^\circ.18$ per orbital revolution [55]. Using our fitting procedure, we get a much bigger precession $3^\circ.26$. Figure 2 in this paper gives theoretically calculated precession per orbital period for hybrid gravity of $\phi_0 - m_\phi$ parameter space. In the future, with much more precise data maybe observation will find smaller value of precession, and using Figure 2 (i.e. the same procedure), we will be able to get again hybrid gravitational parameters m_ϕ and ϕ_0 , hoping that observations will give smaller values. We calculated the map of parameters theoretically for broad range of precession angles. More precise observations probably will change best fit parameters, but procedure for theoretical calculation will be the same.

5. Conclusions

In this paper, the orbit of S2 star around the galactic Centre has been investigated in the framework of the hybrid modified gravity. Using the observed positions of S2 star, we constrained the parameters of hybrid modified gravity. Our simulation results are:

1. the range of values for ϕ_0 parameter, coming from S2 star, is between -0.0009 and -0.0002;
2. the range of m_ϕ is between -0.0034 and -0.0025;
3. precession of S2 star orbit, in the hybrid modified gravity potential, has the same direction as in GR, but the upper limit in magnitude is much bigger than GR.

The above results allow to compare the orbital motion of S2 star in the framework of hybrid gravity with analogous results in other theories. In particular, hybrid gravity can be compared with metric $f(R)$ models, discussed in [29, 36] and with $f(R, \phi)$, discussed in [37]. Also in these papers, the motion of S2 star has been studied according to the effective gravitational potentials achieved in the weak field limit. As discussed above, the main reason to introduce hybrid gravity lies on the fact that models like $f(R)$ gravity (both in metric and Palatini formalism) and $f(R, \phi)$ gravity suffer problems in passing the standard Solar System tests [40, 41]. On the other hand, as reported in [51], hybrid gravity allows to bypass shortcomings deriving from local tests and connect models to galactic dynamics and late time cosmic acceleration. Using S2 star orbits, it is possible to achieve additional constraints at sub-parsec scales and promote this model with respect to other extended gravity approaches.

In particular, ϕ_0 and m_ϕ are the specific parameters of hybrid gravity and differ from $f(R)$ gravity models both in metric and Palatini formalism. In the case of $f(R, \phi)$ gravity, it is possible to achieve a Sanders like potential; the parameter m_ϕ is also present and could have the same value, but the parameter ϕ_0 of hybrid gravity and α of $f(R, \phi)$ differ [37]. The two effective gravitational potentials, in the weak field limit, have similar, but not the same forms at sub-parsec scales.

In conclusion, the comparison of the observed orbits of S2 star and theoretical calculations performed by the hybrid modified gravity model can provide a powerful method for the observational test of the theory, and for observationally discriminating among the different modified gravity models. It seems that hybrid gravity potential is sufficient in addressing the problem of dark matter at galactic scales [51], and it gives indications that alternative theories of gravity could be viable in describing galactic dynamics.

Furthermore, orbital solutions derived from such a potential are in good agreement with the reduced χ^2 deduced for Keplerian orbits. This fact allows to fix the range of variation of ϕ_0 and m_ϕ . The precession of S2 star orbit, obtained for the best fit parameter values (ϕ_0 from -0.0009 to -0.0002 and m_ϕ from -0.0034

to -0.0025), has the positive direction, as in GR, but for these values of parameters, we obtain much larger orbital precession of S2 star in hybrid gravity compared to prediction of GR.

We can conclude that hybrid gravity effective potential is probably the best candidate among the other considered gravity models such as e.g. R^n [29, 38], Yukawa-like [36] and Sanders-like [37] to explain gravitational phenomena at different astronomical scales.

It is important to stress that our comparison with astronomical observations represents only upper bounds for precession angle on the deviation from GR. Although observational data seem to indicate that the S2 star orbit is not Keplerian, the nowadays astrometric limits are not sufficient to unambiguously confirm such a claim. We hope that forthcoming observational data will allow more accurate measurements of stellar positions.

A final remark is due now. From an astrophysical point of view, the main motivation to introduce hybrid gravity is to address the problems of dark matter and dark energy [51]. First of all, we have to say, according to the observations, that dark matter has very negligible effects around the Galactic Centre [57]. Despite of this fact, here we adopted hybrid gravity dynamics only to fit the orbit of S2 star around the Galactic Centre. The interest of the reported results, if confirmed, lies on the fact that hybrid dynamics is independent of the dark issues but can be connected to a fine analysis of geodesic structure. In other words, the further gravitational degrees of freedom, coming from hybrid gravity, contribute to dynamics as soon as orbital analysis related to GR is not sufficient to describe in detail peculiar situations as those around the Galactic Centre. However, in order to better confirm this statement, one needs more precise astronomical data describing stellar dynamics around Galactic Center.

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